Astronomical Approximations. V., VI. By Pliny Earle Chase, LL.D.,
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V. *Cometary Paraboloids.*

Some recent communications to the French Academy, by MM. Gaussin* and Faye,† have led me to re-examine some of my earlier discussions of the influence of projectile forces and perifocal collisions, upon nebular rupture and cosmical nucleation.‡ I have embodied some of the results of this examination in a comparison of my application§ of the general equation

$$x_n = \xi \eta^n \zeta^{n^2} \ldots\ldots (1)$$

with Gaussin's analogous equation ‖

$$a = a\, k^n \ldots\ldots (2)$$

If we let $r_0 = 1$ = Sun's radius; $\xi = 16.164$; $\eta = 1.6252$; $\zeta = 1.013$, equation (1) gives a series of paraboloidal abscissas which represent important cosmical relations.

Bodies falling towards the centre of a cosmical system, from a distance *nd*, acquire the *d*-velocity of revolution, $(\sqrt{gd})$, at the distance $\frac{nd}{n+1}$.

Therefore, $\frac{d_0}{2}$, $\frac{2d_1}{3}$, $\frac{3d_2}{4}$, . . . represent points at which nebular subsidence would tend to produce rupture, with consequent orbital revolution at d_0, d_1, d_2 . . .

In Table I, P represents Stockwell's values for the secular perihelion points of rupture, in units of r_0; A, the values for secular aphelion; T, the theoretical rupturing distances as determined by equation (1).

TABLE I.

			P.	T.	A.
$\frac{1}{2}$ Mercury	=	$\xi\eta^2\zeta^4$	31.91	44 94	51.14
$\frac{1}{2}$ Venus	=	$\xi\eta^3\zeta^9$	72.11	77.91	83.07
$\frac{2}{3}$ Earth	=	$\xi\eta^4\zeta^{16}$	133.34	138.58	152.71
$\frac{3}{4}$ Mars	=	$\xi\eta^5\zeta^{25}$	210.96	252.93	279.38
$\frac{4}{5}$ Ceres	=	$\xi\eta^6\zeta^{36}$	438.66	473.69	511.84
$\frac{5}{6}$ Jupiter	=	$\xi\eta^7\zeta^{49}$	873.59	910.30	986.75
$\frac{6}{7}$ Saturn	=	$\xi\eta^8\zeta^{64}$	1606.19	1795.05	1902.03
$\frac{7}{8}$ Uranus	=	$\xi\eta^9\zeta^{81}$	3319.21	3632.14	3881.96
$\frac{7}{6}$ Neptune	=	$\xi\eta^{10}\zeta^{100}$	7408.37	7541.32	7626.46

* Comptes Rendus t. xc, pp. 518, 593.
† Ib., p. 566.
‡ Proc. Am. Phil. Soc. Vols. ix—xii.
§ Ib. xii, 520.
‖ Loc cit, p. 520. Equation (2) is a special case of equation (1), with $\zeta = 1$.

The perihelion and aphelion values for $\frac{4}{5}$ Ceres were found by taking Newcomb's value for the eccentricity (.077) and mean distance ($2.769\rho_3 = 594.06r_0$). The other values of P and A are computed from Stockwell's elements of planetary distance and secular variation, taking $214.54r_0$ for the value of Earth's semi-axis major (ρ_3).

Dividing the values of T, in Table I, by the respective rupturing coefficients ($\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, . . .), and reducing to units of Earth's semi-axis major, we obtain the values in column C, Table II, for comparison with Gaussin's values, which are given in column G.

TABLE II.

	P.	C.	G.	A.
Mercury	.297	.419	.362	.477
Venus	.672	.726	.623	.774
Earth	.932	.969	1.073	1 068
Mars	1.311	1.572	1.848	1.736
Asteroid	2.132	2.760	3.183	3.954
Jupiter	4.886	5.092	5.483	5.519
Saturn	8.734	9.761	9.445	10.343
Uranus	17.681	19.348	16.269	20.679
Neptune	29.598	30.120	28.025	30.470

The perihelion and aphelion values of the asteroidal belt are represented by the mean distances of (149) and (153). All the other values in columns P and A are computed from Stockwell's elements. Gaussin's approximations to the distances of Venus, Earth, Mars, Uranus, and Neptune, are all outside of the limits of secular excursion for those planets respectively. My own values are all within those limits, and their approximations to the mean values are closer than those of any other similar series that has ever fallen under my notice.

The data for the construction of the cosmothetic paraboloid were derived from considerations of linear oscillation (α), *vis viva* (β,δ), nucleal rotation and synchronous orbital revolution (γ), spherical oscillation (δ), inter-stellar actions and re-actions of projection and attraction (ε), luminous undulation (ζ) and universal nebular subsidence and rupture (η).

α. The focus of paraboloidal action is Sun's centre. The locus of the directrix is also the locus of the linear centre of oscillation of Sun's diameter, referred to Sun's surface ($\frac{2}{3}d - r = \frac{1}{3}r$).

β, γ. The fundamental abscissa (ξ) is $\frac{4}{9}$ L; $\frac{4}{9}$ representing the relative *vis viva* of the linear centre of oscillation ($\frac{2}{3}$), and L being Laplace's limit, or the locus of synchronous rotation and revolution.

δ. The initial ratio ($\eta\zeta^2 = 1.66768$) differs by less than $\frac{1}{100}$ of one per cent. from $\frac{5}{3}$; $\frac{5}{3}$ being the ratio of rupturing *vis viva* to residual *vis viva* of rotation ($1 - .4 = .6$).

ε. The planetary field is geometrically intermediate between the fields of solar nucleation and of stellar projection. There are 9 abscissas between Sun's surface and $\frac{1}{2}$ Mercury ; 9 at loci of theoretical planetary rupture ; 9 between $\frac{7}{6}$ Neptune and the region of the fixed stars.

ζ. The 27th abscissa from Sun's surface, or the 19th from ξ, $(\xi\eta^{19}\zeta^{361})$, is L M $\div$ r_0 ; M being the elastic modulus of light at Sun's surface. (L $=$ 36.36 r_0 ; M $=$ 474600 r_0.)

η. The 28th abscissa from Sun's surface, $(\xi\eta^{20}\zeta^{400})$, is 46352440. Four recent estimates of the distance of α *Centauri* range between 45340000 and 48479500. Searle* cites authorities ranging between 44252000 and 49169000. Newcomb says,† "The mean of all the measures of the parallax of this pair of stars hitherto made, gives 0″.93 as their most probable parallax, corresponding to a distance of 221000 astronomical units." This is equivalent to 47463340 r_0.

It will be readily seen that the elements of the paraboloid, ($\frac{1}{6}$ r_0, L, M), are entirely independent of any observed or theoretical planetary distance. No values can be assumed for those elements, within the limits of possible uncertainty, which will weaken the evidence that the nebula-rupturing position of the several planets, the time of solar rotation, and the interstellar spaces, have been determined by the laws which govern luminous undulation.

VI. Cosmical Determination of Joule's Equivalent.

In estimating heat of dissociation, Pfaundler has shown‡ that the mean should be taken between the temperatures of incipient and of complete dissociation. On this principle, in estimating the temperature of water-crystallization we should have regard to all stages of the expansion in molecular rearrangement, and take the mean (35°.6 F. $=$ 2°C) between the temperatures of greatest density (39°.2 F $=$ 4°C) and of complete crystallization (32° F $=$ 0°C). So long as water continues to condense, its tendencies are centripetal and polar ; while it is expanding, they are centrifugal and equatorial. The thermodynamic relations between heat and work should be shown in the comparative motions and temperatures of polar and equatorial waters, as surely, and with as abundant facilities for accurate measurement, as in the experiments of the laboratory or in the processes of the workshop.

Johnston's Physical Atlas gives 82°.6 F (28°.1C) as the mean temperature of the oceanic warmth-equator. This indicates a polar-equatorial difference of 82°.6 — 35°.6 F $=$ 47 J, or 28°.1 — 2°C $=$ 26.1 calories. The difference in gravitating measure can be readily deduced from the differences of motion. The velocity of equatorial rotation is 1525.78 feet, which represents a fall of $\left(\frac{1525.78}{32.088}\right)^2 \times 16.044$ ft. $=$ 47 J. Hence we find J $=$ 771.816 foot-pounds ; Calorie $=$ 423.44 kilogrammetres.

* Outlines of Astronomy, p. 396.

† Popular Astronomy, p. 208, foot note.

‡ Pogg. Ann., 1867, 131, 603.

The modulus (h) of the velocity of rotation (v_r) is : $h = \frac{v_r^2}{g} = \left(\frac{t_1}{t_0}\right)^2 r$.

Substituting the values of v_r, g, t_1, t_0, we find $h = \frac{1525.782}{32.088} = \left(\frac{5074}{86164}\right)^2 r$ $= 13.741$ miles $= \frac{1}{288.4} r$.

This gives 3949.084 miles for the polar radius, which is $\frac{1}{2}$ mile less than Bessel's estimate, and about $\frac{3}{10}$ of a mile less than Clarke's estimate from the results of the British Ordinance Survey. It accords very closely, however, as we might reasonably have anticipated, with the ellipticity $\left(\frac{1}{288.5}\right)$ as deduced from pendulum experiments. *

Relations of Chemical Affinity to Luminous and Cosmical Energies. By Pliny Earle Chase, LL.D.

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All the principles which I have applied successfully to the discovery of harmonic relations of cosmical mass and density, should be applicable also to the discovery of similar relations of molecular mass and density, and to a consequent extension of our knowledge of chemical physics. The application can be made most properly by chemical experts, but some indications of the methods to be pursued may be acceptable, even though they come from one who makes no claim to any special chemical experience.

In 1833, Sir John Herschel published his remarkable attribution to the energy of the Sun's rays, "of almost every motion which takes place on the surface of the Earth. By its heat are produced all winds, and those disturbances in the electric equilibrium of the atmosphere which give rise to the phenomena of lightning, and probably also to those of terrestrial magnetism and the aurora."†

In 1856, Kohlrausch found that the ratio between the electrostatic and the electromagnetic units was apparently, and perhaps exactly, equivalent to the velocity of light‡ (v_λ).

In 1863, I found that the reaction of gravity to the elasticity and *vis viva* of atmospheric rotation, furnished a simple method for approximately estimating the Sun's distance by means of barometric fluctuations,§ and began a series of studies of cosmical and molecular harmonies which are dependent and consequent upon general kinetic laws.

In the year following,‖ I announced "the discovery of certain new relations between the solar and lunar diurnal variations of magnetic force

* Enc. Brit., 9th edition, vii, 601.
† Outlines of Astronomy, § 399.
‡ Pogg. Ann.
§ Proc. Am. Phil. Soc., ix, 287.
‖ Ib. ix, 425.